Maths Challenge - Week 300 – Solutions (6 years of puzzling)

Welcome to week 300 of our weekly maths challenge, with problems and puzzles posed by David Browning, Rod Marshall, Ian Stewart, Annie Stothers and the u3a Maths and Stats Subject Adviser - David Martin. If you would like to share your ideas on how to solve these puzzles please join our learning forum or discuss within your u3a and interest group. Check back each week for the solutions and let us know how you get on by contacting the u3a office. New maths puzzles will go up onto the website every Thursday.

Question 1.

Brenda thought of a number, doubled it, subtracted forty, divided by seven and added twenty resulting in a third of the number. What number did Brenda think of?

Solution

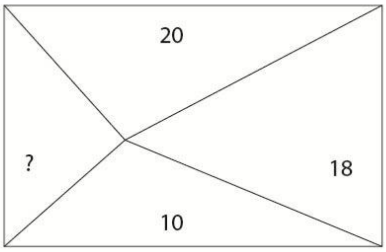
Suppose that Brenda thought of the number x

Then, $(2x – 40)/7 + 20 = x/3$

Multiplying through by 21 we have $6x - 120 + 420 = 7x$ so, $x = 300$

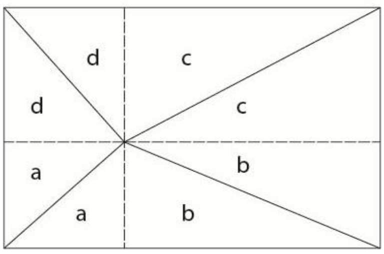
Brenda was thinking of the number 300.

Question 2.



This rectangle has been divided into four triangular areas. The area of three of these is shown in the diagram; what is the area of the fourth?

Solution



Dividing each triangle into two to produce four pairs of triangles of areas a, b, c and d, we require a + d.

We know that $a + b = 10$, $b + c = 18$ and $c + d = 20$

So, $a + d = (a + b) – (b + c) + (c + d) = 10 - 18 + 20 = 12$

Question 3.

Including 1 and the number itself, how many numbers smaller than 300 have exactly six factors?

Solution

With thanks to Laurence for correcting our original sample solution.

If a number N has a unique prime factor p, then to have six factors we must have N = p^5, giving the factors 1, p, p^2, p^3, p^4 and p^5. As $3^5 = 243 < 300$, but $5^5 = 3125 > 500$, both p = 2 and p = 3 satisfy the required condition – **2** numbers.

If N has three unique prime factors p, q and r, then it must have at least eight factors: 1, p, q, r, pq, pr, qr and pqr, so we can rule out this possibility.

This leaves the case of two distinct prime factors, p and q, to consider.

If N = pq then the factors are 1, p, q and pq, but we need N to have six factors.

If N = p^2q then the factors are 1, p, q, pq, p^2 and p^2q – a total of 6 as required. So, we need to search for numbers less than 300 of the form p^2q, where p and q are distinct primes.

If p = 2, q ≤ 75 as $300/2^2 = 75$, which gives **20** numbers: 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71 and 73 (2 can be excluded as p and q must be distinct).

If p = 3, q ≤ 33.3…, which gives **10** numbers: 2, 5, 7, 11, 13, 17, 19, 23, 29 and 31.

If p = 5, q ≤ 12, which gives **4** numbers: 2, 3, 7 and 11.

If p = 7, q ≤ 6.1…, which gives **3** numbers: 2, 3 and 5.

If p = 11, q ≤ 2.4…, which gives **1** number: 2.

If p ≥ 13, q ≤ 1.7…, which is impossible.

So, in total there are 2 + 20 + 10 + 4 + 3 + 1 = 40 such numbers.

Question 4.

If each letter of the alphabet is converted into a number by first multiplying the letter's position in the alphabet by an integer, A, and then adding integer B, the sum of the numbers for the letters in LEARN is 1135 and in LAUGH is 1111. What is the sum of the numbers for the letters in LIVE?

Solution

The sum of the numbers for the letters in LEARN will be A x (sum of the positions of the letters in LEARN) + 5B. This is 50A + 5B = 1135. Similarly, the sum of the numbers for the letters in LAUGH is 49A + 5B = 1111. It follows that A = 24 and B = -13. The sum of the numbers for the letters in LIVE is then 1100.