Maths Challenge - Week 302 – Solutions

Welcome to week 302 of our weekly maths challenge, with problems and puzzles posed by David Browning, Rod Marshall, Ian Stewart, Annie Stothers and the u3a Maths and Stats Subject Adviser - David Martin. If you would like to share your ideas on how to solve these puzzles please join our learning forum or discuss within your u3a and interest group. Check back each week for the solutions and let us know how you get on by contacting the u3a office. New maths puzzles will go up onto the website every Thursday.

Question 1.

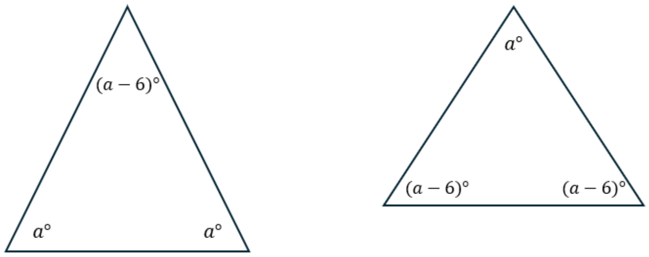
What is the largest possible angle in an isosceles triangle, in which the difference between the largest and smallest angles is $6°$?

Solution

Two angles in an isosceles triangle are equal.

Let the largest angle be $a°$, and then the smallest angle would be $(a-6)°$.

There are two possibilities as shown in the diagrams below.



In the first we have $3a - 6 = 180$, so $a = 62°$.

In the second we have $3a - 12 = 180$, so $a = 64°$.

Therefore, the largest possible angle is $64°$.

Question 2.

The eight-digit number "$aaaabbbb$", where a and b are digits, is a multiple of 45. What are the possible values of a, $a \neq 0$?

Solution

Every multiple of 45 is a multiple of both 5 and 9 so, applying the usual rules of divisibility by 5 and 9 to the number, we can deduce that:

$b = 0$ or $b = 5$ and that $4a + 4b$ is a multiple of 9.

In the case $b = 0$, $4a$ and so a is a multiple of 9, hence $a = 9$.

In the case $b = 5$, $4a + 20$ and so $a + 5$ is a multiple of 9, hence $a = 4$.

There are two possible numbers: 99 990 000 and 44 445 555.

Question 3.

How many three-digit whole numbers are divisible by 3 or 4?

Solution

There are 900 three-digit whole numbers, from 100 to 999 inclusive.

900 is divisible by 3, so we can group these 900 numbers into 300 sets of three consecutive numbers. There will be exactly one number divisible by 3 in every set, so 300 three-digit whole numbers can be divided by 3.

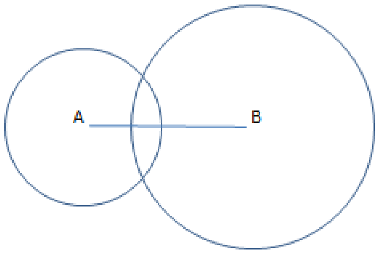
Similarly, 900 is divisible by 4, so we can group the 900 numbers into 225 sets of four consecutive numbers. There will be one number divisible by 4 in every set, so 225 three-digit whole numbers can be divided by 4.

If we combine these two sets to get 525 numbers however, we will double-count the numbers that are divisible by *both* 3 and 4, i.e. numbers that are a multiple of 3 x 4 = 12. Splitting the original 900 numbers into 75 sets of 12 consecutive numbers shows that 75 three-digit whole numbers can be divided by 12.

Hence 300 + 225 – 75 = 450 three-digit whole numbers are divisible by 3 or 4.

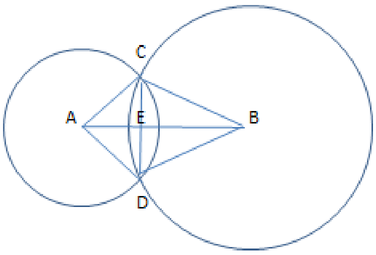
Question 4.

In the diagram below, the smaller circle has a radius of 20 cm and the larger circle has a radius of 25 cm. The centres of the circles are marked as A and B and the distance between A and B is 40 cm. What is the area of the overlap of the two circles?



Solution

In the diagram below, the points at which the circles intersect are labelled C and D. AB intersects CD at point E.



If the length of AE is d cm, the length of BE is (40 – d) cm. As the length of AC is 20 cm, by Pythagoras the length of CE is $\sqrt{(400 - d^2)}$. As the length of BC is 25 cm, CE can also be calculated as $\sqrt{(625 - (40 - d)^2)}$. It follows that $400 - d^2 = 625 - (1600 - 80d + d^2)$ which gives d = 1375/80 = 17.1875 cm and the length of CE is 10.22692 cm. Due to symmetry, CE and DE are the same length.

The area of triangle ACD is then 17.1875 x 10.22692 = 175.77519 cm^2. We can now calculate the area of the segment of the smaller circle which is to the right of chord CD by calculating the area of the sector between AC and AD and subtracting the area of triangle ACD. Cos (∠CAE) = 17.1875/20 so ∠CAE = 30.75352° so ∠CAD = 61.50704°. The area of the sector ACD is then (61.50704/360) x 400π = 214.70007 cm^2. The area of the segment of the smaller circle to the right of chord CD is 214.70007 - 175.77519 = 38.92488 cm^2.

The length of BE is 40 – 17.1875 = 22.8125 cm. The area of triangle BCD is then 22.8125 x 10.22692 = 233.30161 cm^2. Cos (∠CBE) = 22.8125/25 = 0.9125 so ∠CBE = 24.14685° and so ∠CBD = 48.2937°. The area of the sector BCD is (48.2937/360) x 625π = 263.40127 cm^2. The area of the segment of the larger circle to the left of chord CD is 263.40127 – 233.30161 = 30.09966 cm^2.

The area of overlap of the two circles is then 38.92488 + 30.09966 = 69.02454 cm^2 = 69.02 cm^2 to two decimal places.