

Maths Challenge - Week 296 – Solutions

Welcome to week 296 of our weekly maths challenge, with problems and puzzles posed by David Browning, Rod Marshall, Ian Stewart, Annie Stothers and the [u3a Maths and Stats Subject Adviser](#) - David Martin. If you would like to share your ideas on how to solve these puzzles please join our [learning forum](#) or discuss within your u3a and interest group. Check back each week for the solutions and let us know how you get on by contacting the [u3a office](#). New maths puzzles will go up onto the website every Thursday.

Question 1.

Jan's purse contains six coins; 1p, 2p, 5p, 10p, 20p and 50p. What coins would she need to add to be able to make up any amount from 1p to 99p?

Solution

For amounts less than £1, with the six coins 1p, 2p, 5p, 10p, 20p and 50p, she could not make up e.g., 4p, 9p, 40p and 90p. Adding either another 1p or 2p and either a 10p or 20p she could also make up those amounts and all the amounts from 1p to 99p.

Question 2.

Solve $\sqrt{x+9} + \sqrt{16-x} = 7$

Solution

Square both sides: $x + 9 + 16 - x + 2\sqrt{(x+9)(16-x)} = 49$

Simplify: $25 + 2\sqrt{(x+9)(16-x)} = 49$; $\sqrt{(x+9)(16-x)} = 12$

Square again: $(x+9)(16-x) = 144$

Expand: $144 + 7x - x^2 = 144$; $x(7-x) = 0$

So, $x = 0$ or $x = 7$

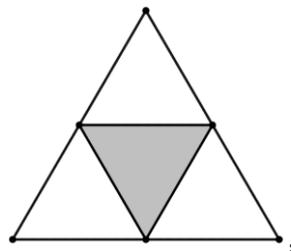
Question 3.

If you connect the midpoints of adjacent sides of a regular polygon, you get a smaller regular polygon with the same number of sides. What proportion of the area of the larger polygon is the smaller polygon when the number of sides is:

- a) 3?
- b) 4?
- c) 6?

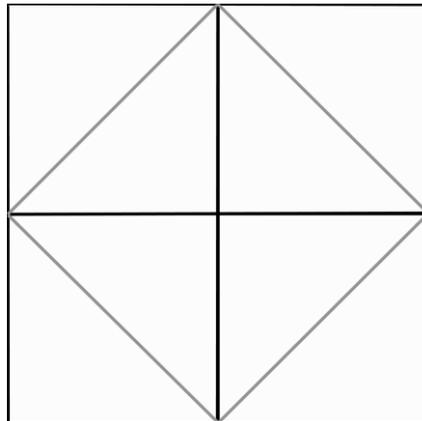
Solution

(a) An equilateral triangle is divided into four congruent equilateral triangles when the midpoints of its sides are joined:



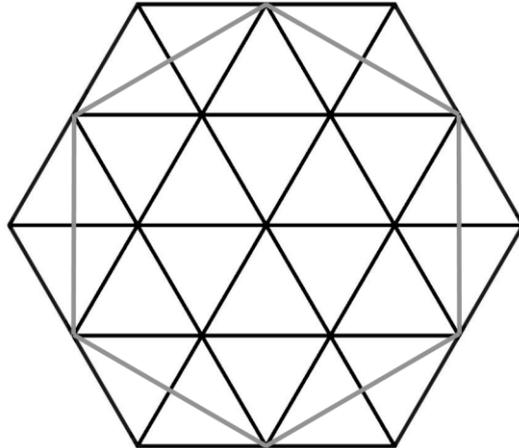
It is then easy to see that the area of the smaller (shaded) triangle is $\frac{1}{4}$ of the area of the original triangle.

(b) A square can be divided into four smaller, equal squares by joining the midpoints of the opposite sides, as shown in black in the diagram:



When the square formed by joining the midpoints of the larger square's sides is drawn - shown in grey in the diagram - it covers half of each of the four smaller squares, so the proportion of the area of the original square taken up by the smaller one is $\frac{1}{2}$.

(c) A regular hexagon can be divided into 24 identical equilateral triangles by joining the opposite vertices and dividing the resulting equilateral triangles into four smaller equilateral triangles as in (a). When the smaller hexagon formed by joining the



midpoints of the original hexagon's sides is drawn (see diagram), it covers half of each of the 12 smaller triangles around the edge of the outer hexagon, and all 12 of the triangles in the centre of the hexagon. The proportion of the area of the larger hexagon taken up by the smaller one is therefore $(\frac{12}{2} + 12)/24 = \frac{18}{24} = \frac{3}{4}$.

Question 4.

John is going to walk at a constant speed of 1 m/s to a bench which is 600 metres from him. His grandson Peter prefers to run and he decides that, when John starts to walk towards to the bench, he will run to the bench and then run back to John. On his first return trip, Peter runs at a speed of 5 m/s. He then continues running to the bench and then back to John but his speed on the second return trip is 4 m/s, on the third return trip it is 3 m/s, on the fourth return trip it is 2 m/s and he then walks at 1 m/s with John until they both reach the bench. How far will Peter have run or walked when he and John reach the bench?

Solution

On his first return trip, if A is the distance that John has walked when Peter returns to him, John will have walked for A seconds and Peter will have run for $(1200 - A)/5$ seconds. As these times must be the same, $A = (1200 - A)/5$ so $A = 200$ m. Peter will have run 1000 m on his first return trip.

The distance to the bench is now 400 m so using the same logic as above, the distance, B, which John walks during Peter's second return trip can be found from the equation $B = (800 - B)/4$ which gives $B = 160$ m. Peter will have run 640 m on his second return trip.

The distance to the bench is now 240 m. The distance, C, which John walks during Peter's third return trip can be found from $C = (480 - C)/3$ which gives $C = 120$ m. Peter will have run 360 m on his third return trip.

The distance to the bench is now 120 m. The distance, D, which John walks during Peter's fourth return trip can be found from $D = (240 - D)/2$ which gives $D = 80$ m. Peter will have run 160 m on his fourth return trip.

The distance to the bench is now 40 m and Peter walks this distance with John.

Total distance run or walked by Peter is 2200 m.