Maths Challenge - Week 297 – Solutions

Welcome to week 297 of our weekly maths challenge, with problems and puzzles posed by David Browning, Rod Marshall, Ian Stewart, Annie Stothers and the u3a Maths and Stats Subject Adviser - David Martin. If you would like to share your ideas on how to solve these puzzles please join our learning forum or discuss within your u3a and interest group. Check back each week for the solutions and let us know how you get on by contacting the u3a office. New maths puzzles will go up onto the website every Thursday.

Question 1.

On a certain planet, a year has 360 days, arranged into 12 months of 30 days each. A week has 6 days (Monday to Saturday). The first day of Year 1 is a Monday. What day of the week is the first day of Year 25?

Solution

Each month has exactly 30/6 = 5 weeks so, the first day of each month is also a Monday as is the first day of Year 25.

Question 2.

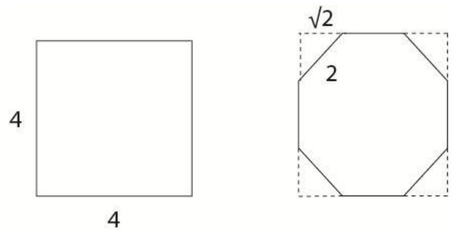
Bill has eight two-metre sections of fencing and is trying to decide whether to arrange them into a square or an octagon to enclose the largest area. Which will give him the largest enclosed area and by what percentage does this benefit over the other arrangement?

Solution

The square gives an enclosed area of $16m^2$

An octagon gives an enclosed area of $(2+2\sqrt{2})(2+2\sqrt{2}) - 2\sqrt{2}\sqrt{2} = 8(1+\sqrt{2})$

This is greater than 16 by $\frac{8(-1+\sqrt{2})}{16} \times 100\% = 20.71\%$ to 2 decimal places.



Question 3.

(a) In any calendar year, what is the maximum number of times the 13th of a month can fall on a Friday?
(b) Is it possible for a year to have no Friday the 13ths?

Solution

We will approach both parts working modulo 7, as there are seven days in a week. Counting modulo n means finding the remainder when a number is divided by a given modulus n. This procedure is usually denoted "mod n". For example, 23 mod 5 = 3, because 23 divided by 5 has a remainder of 3.

(a) If we call 13th January day 0, then 13th February is day 0 + 31 = 3 mod 7. If the year is not a leap year, the sequence of 13ths for the full year is:

0, 3, 3, 6, 1, 4, 6, 2, 5, 0, 3, 5 mod 7.

The number 3 appears three times in this sequence; no other number appears more than twice. This means that, in a non-leap year, there can be at most three Friday the 13ths: if February 13th is a Friday then the 13ths of March and November will be too (which is the case for 2026).

In a leap year, the sequence for the year is 0, 3, 4, 0, 2, 5, 0, 3, 6, 1, 4, 6. This time the number 0 appears three times while no other number appears more than twice, so again there can be at most three Friday the 13ths. If January 13th is a Friday, April and July will be too.

(b) By inspection, both series in the solution to part (a) include every number between 0 and 6 at least once. This means every year will have at least one Friday the 13th.

Question 4.

A builder is preparing to construct a brick wall of height 1.93 m between two concrete posts which are separated by a distance of 5.2 m. The wall will rest on a concrete foundation and odd and even rows will be commenced using full and half bricks respectively. The selected bricks have dimensions of 215 x 102.5 x 65 mm (length x width x height) and do not have frogs (recesses). The half-bricks will be shortened to a length of 102.5 mm. If the mortar thickness is 10 ± 1 mm, and whole bricks are to be used throughout with the exception of the ends of the even rows, what will be the minimum number of 10 kg tubs of pre-prepared wet mortar which will be required assuming the mortar density is 2000 kg m^{-3}?

Solution

Each row will consist of a mortar base plus a line of bricks bonded with mortar. The number of rows N_{rows} can be estimated in terms of the height of the wall h_w, the height of the brick h_b and the nominal horizontal thickness of the mortar m_t.

$$N_{rows} = \frac{h_w}{h_b + m_t} = \frac{1.93}{0.065 + 0.01} = 25.733 \text{ to 3 decimal places.}$$

Rounding up N_{rows} to 26, rearranging and solving for m_t,

$$m_t = \frac{h_w}{26} - h_b = 9.23 \text{ mm to 2 decimal places.}$$

Therefore, the wall height can be achieved using 26 rows of bricks and a horizontal mortar thickness of 9.23 mm which is within the required tolerance.

The number of whole bricks per odd row N_{wo} can be estimated from the length of the wall l_w, the length of the brick l_b and the nominal vertical thickness of the mortar m_{to} noting that each end of the wall adjoining the concrete posts will require a mortar bond.

$$N_{wo} = \frac{l_w - m_{to}}{l_b + m_{to}} = \frac{5.2 - 0.01}{0.215 + 0.01} = 23.067 \text{ to 3 decimal places.}$$

Rounding down N_{wo} to 23, rearranging and solving for m_{to},

$$m_{to} = \frac{l_w - 23\, l_b}{24} = 10.63 \text{ mm to 2 decimal places.}$$

Therefore, for odd row numbers, the wall length can be achieved using 23 whole bricks and a vertical mortar thickness of 10.63 mm which is within the required tolerance.

For even row numbers, the vertical mortar thickness m_{te} can be calculated from the length of the wall l_w, the lengths of the brick l_b and half brick l_{hb} noting that there are 22 whole bricks and 2 half bricks and an additional mortar bond compared to the odd row:

$$m_{te} = \frac{l_w - 22\, l_b - 2\, l_{hb}}{25} = 10.60 \text{ mm to 2 decimal places.}$$

Therefore, even row numbers can be achieved with 22 whole bricks, 2 half bricks and a vertical mortar thickness of 10.60 mm which is within the required tolerance.

The total volume of the bricks in the wall V_{bt} can be calculated in terms of the volumes of the individual whole bricks V_{bw} and the individual half-bricks V_{bh}:

$$V_{bt} = (13 \text{ x } 23 + 13 \text{ x } 22)\, V_{bw} + (13 \text{ x } 2)\, V_{bh}$$

$$V_{bw} = 0.215 \text{ x } 0.1025 \text{ x } 0.065$$

$$V_{bh} = 0.1025 \text{ x } 0.1025 \text{ x } 0.065$$

$$V_{bt} = 585\, (0.215 \text{ x } 0.1025 \text{ x } 0.065) + 26\, (0.1025 \text{ x } 0.1025 \text{ x } 0.065)$$

$V_{bt} = 0.83798 + 0.01776$ = 0.8557 m^3 to 4 decimal places.

The volume of the wall V_w can be calculated as:

$V_w = 1.93\ x\ 5.20\ x\ 0.1025 = 1.0287$ m^3 to 4 decimal places.

The volume of mortar V_w can be calculated as:

$V_m = V_w - V_{bt} = 0.173$ m^3 to 3 decimal places.

The total weight of mortar W_m can be calculated from the volume V_m and the density D_m:

$W_m = D_m V_m$ = 2000 x 0.173 = 346 kg

Therefore, a minimum of 35 tubs of 10 kg will be required.