Maths Challenge - Week 299 – Solutions

Welcome to week 299 of our weekly maths challenge, with problems and puzzles posed by David Browning, Rod Marshall, Ian Stewart, Annie Stothers and the u3a Maths and Stats Subject Adviser - David Martin. If you would like to share your ideas on how to solve these puzzles please join our learning forum or discuss within your u3a and interest group. Check back each week for the solutions and let us know how you get on by contacting the u3a office. New maths puzzles will go up onto the website every Thursday.

Question 1.

Three numbers have a product of 195 and a sum of 29. What are these three numbers?

Solution

195 = 3 x 5 x 13 = 1 x 13 x 15 = 1 x 3 x 65 = 1 x 5 x 39 with sums 21, 29, 69 and 45

Hence, the three numbers are 1, 13 and 15

Question 2.

A man is three times as old as his son. In 12 years, he will be twice as old. How old are they now?

Solution

Suppose that the son's age is x, then the father's age is $3x$.

In 12 years, $3x + 12 = 2(x + 12) = 2x + 24$ so, $x = 12$

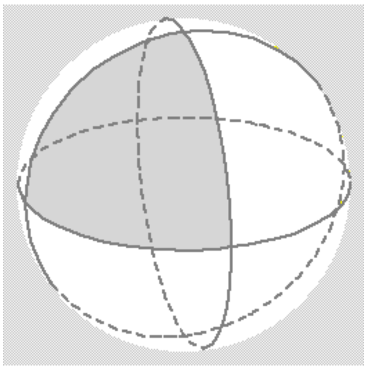
The son is 12 and the father is $3 \times 12 = 36$

Question 3.

Nine points are selected on a sphere of radius 1. Show that there must be a pair of points not more than √2 units apart.

Solution

The surface of the sphere can be split into eight equal regions using three great circles meeting at right angles to each other. This is illustrated below, with one of the eight surface regions shaded.

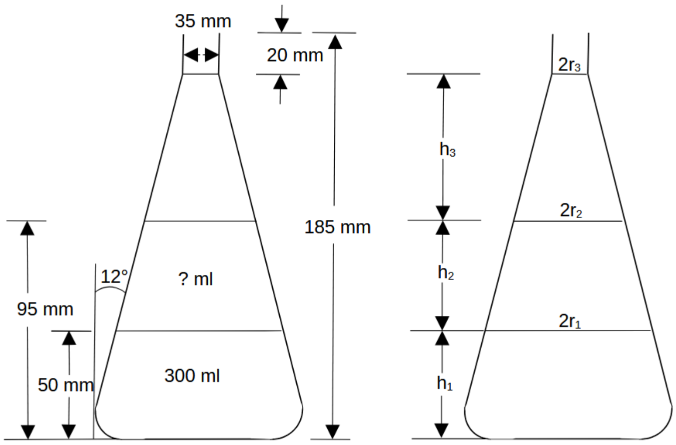


As there are eight regions and nine selected points, (at least) one of the regions must contain two or more of the nine points. The furthest distance these two points can be apart, if we measure in a straight line through the interior of the sphere, is when they are at places where the circles intersect. It can be calculated using Pythagoras' theorem for a triangle with a right-angle between two radii of the sphere going from its centre to each of the two intersection points. Since the radius is 1, this distance is $\sqrt{(1^2 + 1^2)}$ units = $\sqrt{2}$ units.

Question 4.

A laboratory type conical flask has a total height of 185 mm including a neck of length 20 mm and a side taper angle of 12º. The internal diameter of the neck is 35 mm. When an initial quantity of 300 ml of liquid is poured into the flask, the height of the liquid is 50 mm. When a further amount of liquid is added, the height increases to 95 mm. What is the total volume of liquid in the flask?

Solution



From the dimensions provided, the values of the heights h_1, h_2 and h_3 and the internal radii r_1, r_2, and r_3 can be inferred:

$$h_1 = 50 \text{ mm}$$

$$h_2 = 95 - h_1 = 45 \text{ mm}$$

$$h_3 = 185 - 20 - h_1 - h_2 = 70 \text{ mm}$$

$$r_3 = \frac{35}{2} = 17.5 \text{ mm}$$

$$\frac{r_1 - r_3}{h_2 + h_3} = tan(12^o)$$

$r_1 = 17.5 + 115 \text{ x } 0.21256 = 41.944 \text{ mm}$ to 3 decimal places.

$$\frac{r_2 - r_3}{h_3} = tan(12^o)$$

$r_2 = 17.5 + 70 \text{ x } 0.21256 = 32.379 \text{ mm}$ to 3 decimal places

The increase in volume represented by h_2 can be modelled in terms of the standard formula for the volume of a truncated cone (frustum).

$$V = \frac{\pi h_2}{3}({r_1}^2 + r_1 r_2 + {r_2}^2)$$

Substituting for h_2, r_1 and r_2:

$V = \frac{\pi \text{ x } 45}{3}(1759.299 + 1358.105 + 1048.400) = 196309$ mm$^3 =$ 196.309 ml

Therefore, the total volume of liquid in the flask is 496 ml to the nearest millilitre.