Maths Challenge - Week 298 – Solutions

Welcome to week 298 of our weekly maths challenge, with problems and puzzles posed by David Browning, Rod Marshall, Ian Stewart, Annie Stothers and the u3a Maths and Stats Subject Adviser - David Martin. If you would like to share your ideas on how to solve these puzzles please join our learning forum or discuss within your u3a and interest group. Check back each week for the solutions and let us know how you get on by contacting the u3a office. New maths puzzles will go up onto the website every Thursday.

Question 1.

Find a two-digit number such that:

- The sum of its digits is **10**
- Reversing the digits increases the number by **36**

Solution

Let the digits of the number be ab

Then, $10b + a = (10a + b) + 36$ i.e., $9b = 9a + 36$ or, $b = a + 4$

$10 = a + b = a + (a + 4) = 2a + 4$ so, $a = 3,\ b = 7$

The two-digit number is 37

Question 2.

Two fair six-sided dice are thrown. What is the probability that the sum is a multiple of 3?

Solution

The total number of outcomes for a pair of dice is $6 \times 6 = 36$

The outcome sums divisible by 3 are

3 (2 ways), 6 (5 ways), 9 (4 ways), 12 (1 way), a total of 12 ways

The probability that the sum of the pair of dice is a multiple of 3 is $\frac{12}{36} = \frac{1}{3}$

Question 3.

A set of real numbers {a, b, c, d} satisfies the following equations:

(1) $a + b + c = 2$

(2) $a^2 + bc = 1$

(3) $a(b + c) = d^2 + 1$

Find all possible values for the set of numbers.

Solution

From (1), $b + c = 2 – a$

Substituting this into (3) gives $a(2 – a) = d^2 + 1$

This rearranges to $– a^2 + 2a – 1 = d^2$, or $(a – 1)^2 + d^2 = 0$

As the squares of both (a – 1) and d are non-negative, this is only possible if a = 1 and d = 0. Hence from (1) we have b + c = 1 and from (2) we have bc = 0. Taken together, these two equations mean either:

b = 0 and c = 1 or, c = 0 and b = 1.

So, there are just two possible sets of values for {a, b, c, d}, namely

{1, 1, 0, 0} and {1, 0, 1, 0}.

Question 4.

Starting with a number, add 4, then calculate the positive square root of the result and add 10, then multiply the result by 13 and then subtract 20. If the final result is the same as the starting number, what is that number?

Solution

If the starting number is n, the information in the question means that:

$$\left(\sqrt{n + 4} + 10\right) \times 13 - 20 = n$$

$$n - 110 = 13\sqrt{n + 4}$$

$$(n - 110)^2 = 169(n + 4)$$

$$n^2 - 220n + 12100 = 169n + 676$$

$$n^2 - 389n + 11424 = 0$$

$$(n - 357)(n - 32) = 0$$

So, possible values of n are 32 and 357.
However, if the starting number is 32, the final result will only be 32 if the negative square root of (32 + 4) is used. The only solution where the positive square root is calculated is where 357 is the starting number.