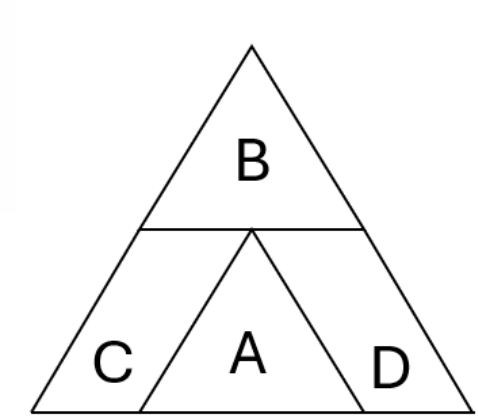
Maths Challenge - Week 293 – Solutions

Welcome to week 293 of our weekly maths challenge, with problems and puzzles posed by David Browning, Rod Marshall, Ian Stewart, Annie Stothers and the u3a Maths and Stats Subject Adviser - David Martin. If you would like to share your ideas on how to solve these puzzles please join our learning forum or discuss within your u3a and interest group. Check back each week for the solutions and let us know how you get on by contacting the u3a office. New maths puzzles will go up onto the website every Thursday.

Question 1.

In the diagram, the large equilateral triangle is divided into two identical equilateral triangles A and B, and two parallelograms C and D which are mirror images of each other. What is the ratio of area D to area B?



Solution

Let us call the large triangle X.

Since triangles A and B are congruent, they have the same height, which is half the height of X.

Therefore, the area of each of A and B is a quarter of the area of X.

Parallelograms C and D together form the other half and thus each occupies a quarter of the area of X.

So, D and B are equal in area with area ratio 1:1

Question 2.

In a sequence of six numbers, every term after the second term is the sum of the previous two terms. Also, the last term is four times the first term, and the sum of all six terms is 13. What are the first two terms?

Solution

Letting the first two terms be x and y, the sequence can be written as

x, y, x + y, x + 2y, 2x + 3y, 3x + 5y

The last term is four times the first term and so, $3x + 5y = 4x \Rightarrow 5y = x$

The sum of all six terms is 13 so, $8x + 12y = 13$

As $5y = x$, $40y + 12y = 13$

$52y = 13$

$y = 13/52 = 1/4$

$x = 5y = 5/4$

The first two terms are 5/4 and 1/4.

Question 3.

(a) What is the maximum possible sum of the visible faces if you stack n dice on top of each other on a table to form a tower?

(b) What is the maximum possible sum of the visible faces if you use eight dice to make a 2 x 2 x 2 cube on the table?

Solution

(a) All the dice in the tower will have four visible faces, with the exception of the one at the top which will have five. As the faces on opposite sides of dice always add to 7, this means the faces of the dice, excluding the uppermost face on the top one for the moment, will add to 14n. By making sure the top face is showing a 6, the maximum possible sum of the visible faces is then seen to be 14n + 6.

(b) Consider the four dice on the table. Each of them will have only two visible faces. The maximum possible sum will be achieved by having the 6s and 5s facing outwards, giving 4 x (6 + 5) = 44.
On the upper half, all four dice will have three visible faces. To maximise the sum, we want each of the dice to be showing 4, 5 and 6, giving a total of 4 x (4 + 5 + 6) = 60.

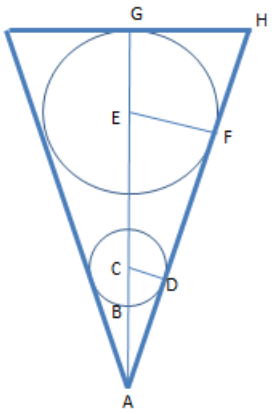
The overall total of the visible faces therefore has a maximum value of 44 + 60 = 104.

Question 4.

A ball of diameter 2 cm is placed into an inverted cone. A ball of diameter 6 cm is then placed into the cone. The two balls do not touch each other and the shortest distance between the two balls is 1 cm. A lid is then placed on the cone, and this lid touches the top of the larger ball. What percentage of the volume of the cone is occupied by the two balls?

Solution

The diagram below shows the side view of the two balls inside the cone.



As the two balls must be touching the side of the cone, line AH is a tangent to the circles representing the two balls. In the diagram, EF is a radius of the large circle and angle AFE is 90°. Similarly, CD is the radius of the small circle and angle ADC is 90°.

AG, the height of the cone, equals the diameters of the two balls, i.e. 2cm + 6cm = 8cm, plus the 1cm gap between them, plus AB. Thus AG = AB + 9cm.

As the angles of triangle ADC are the same as the angles of AFE, the triangles are similar and so AC/CD = AE/EF. This means that (AB + 1)/1 = (AB + 6)/3 so AB = 1.5 cm. The height of the triangle, AG, is therefore 10.5 cm.

sin (CAD) = CD/AC = 0.4 so angle CAD = 23.5782° to 4 decimal places. Angle GAH is the same as angle CAD so GH = AG x tan (GAH) = 10.5 x tan 23.5782° = 4.583 cm to 3 decimal places.

The volume of a sphere is $4\pi r^3/3$ where r is the radius of the sphere and the volume of a cone is $\frac{1}{3}\pi r^2 h$ where r is the radius of the base of the cone and h is its height. The percentage of the volume occupied by the two balls is therefore $(112\pi/3)/(\pi \times 4.583^2 \times 10.5/3) = 0.508$. So 51%, to two significant figures, of the volume of the cone is occupied by the balls.