Maths Challenge - Week 294 – Solutions

Welcome to week 294 of our weekly maths challenge, with problems and puzzles posed by David Browning, Rod Marshall, Ian Stewart, Annie Stothers and the u3a Maths and Stats Subject Adviser - David Martin. If you would like to share your ideas on how to solve these puzzles please join our learning forum or discuss within your u3a and interest group. Check back each week for the solutions and let us know how you get on by contacting the u3a office. New maths puzzles will go up onto the website every Thursday.

Question 1.

Jane has a bag of sweets. She gives one third of them to Simon and she gives five to Anne. She then eats one and gives one quarter of her remaining sweets to Paul. If Jane now has six sweets, how many sweets did she originally have?

Solution

Working backwards, the original number of sweets was ((6 x 4/3) + 1 + 5) x 3/2 = 21.

Question 2.

Ian, Jane and Kevin played each other at chess. Ian won half of the games and if Kevin had won, rather than lost, one of his games against Ian, they would all have won a third of the games. How many games did they play?

Solution

The number of games must be a multiple of both 2 and 3 for both half and third of the games to be a whole number.

If the number of games is 6n then Ian won 3n games and 3n – 1 = 2n i.e., n = 1 so, they played 6 games.

Question 3.

Two whole numbers x and y satisfy this equation:

$x^2 - y^2 + 2(x + 2y) = 2026$

Given that neither x nor y is prime, find x and y.

Solution

The equation can be rewritten as the difference of two squares:

$(x + 1)^2 - (y - 2)^2 = 2026 - 3$

$\Rightarrow (x + 1 + y - 2)(x + 1 - (y - 2)) = 2023$

$\Rightarrow (x + y - 1)(x - y + 3) = 2023$

As x and y are whole numbers, we need factor pairs of 2023 that could be the values of $(x + y - 1)$ and $(x - y + 3)$. As $2023 = 7 \times 17 \times 17$, the only possibilities are 1×2023, 7×289 or 17×119.

$x + y - 1 > x - y + 3$ for any $y > 2$, so consider the cases $y = 1$ or $y = 2$.

$y = 1$ would give $(x + 2)(x + 2) = 2023$, and $y = 2$ would make $(x + 1)(x + 1) = 2023$. Neither is possible since 2023 is not a square number.

So we must now consider the following simultaneous equations:

(i) $x + y - 1 = 2023$ and $x - y + 3 = 1$

(ii) $x + y - 1 = 289$ and $x - y + 3 = 7$

(iii) $x + y - 1 = 119$ and $x - y + 3 = 17$

Adding the equations to find x and subtracting them to find y leads to:

(i) $x = 1011$ and $y = 1013$

(ii) $x = 147$ and $y = 143$

(iii) $x = 67$ and $y = 53$

The question states neither x nor y is prime, so we can discard (i) as 1013 is prime and (iii) as both 67 and 53 are prime.

But both 147 and 143 are composite ($147 = 3 \times 49$ and $143 = 11 \times 13$), so (ii) gives us the required values.

Question 4.

The sum of five different whole numbers is 200, the sum of the squares of the five numbers is 8314 and the difference between the largest and smallest numbers is 19. If two of the five numbers are square numbers, what are the five numbers?

Solution

If S is the smallest number and L is the largest number, the maximum value for L will occur when the other three numbers are S + 1, S + 2 and S + 3. This means that $S + (S + 1) + (S + 2) + (S + 3) + (S + 19) = 200$ which gives S = 35. The maximum value for L is therefore 35 + 19 = 54 and the maximum value for S is 35.

The minimum value for L will occur when the other three numbers are L – 1, L – 2 and L – 3. This means that $(L - 19) + (L - 3) + (L - 2) + (L - 1) + L = 200$ which gives L = 45. The minimum value for L is therefore 45 and the minimum value for S is 26.

These minimum and maximum values imply that the two square numbers must be 36 and 49.

If 49 is the largest of the five numbers, the smallest will be 30. If A and B are the two unidentified numbers then
$A + B = 200 - 49 - 36 - 30 = 85$ and
$A^2 + B^2 = 8314 - 49^2 - 36^2 - 30^2 = 3717$.

As $B = 85 - A$, $A^2 + (85 - A)^2 = 3717$ which simplifies to give $2A^2 - 170A + 3508 = 0$. As this quadratic equation has no whole number roots, 49 cannot be the largest of the five numbers.

If L is the largest number and M is the unidentified number, then
$(L - 19) + 36 + 49 + M + L = 200$ so $2L + M = 200 - 36 - 49 + 19 = 134$.

In addition, $(L - 19)^2 + (134 - 2L)^2 + L^2 = 8314 - 49^2 - 36^2 = 4617$. This equation can be simplified to give $6L^2 - 574L + 13700 = 0$ which can be factorised to give $(2L - 100)(3L - 137) = 0$ so L = 50 or 137/3. As L is a whole number, L = 50 and M = 34.

The five whole numbers are then 31, 34, 36, 49 and 50.