**Maths Challenge - Week 295 – Solutions**

Welcome to week 295 of our weekly maths challenge, with problems and puzzles posed by David Browning, Rod Marshall, Ian Stewart, Annie Stothers and the u3a Maths and Stats Subject Adviser - David Martin. If you would like to share your ideas on how to solve these puzzles please join our learning forum or discuss within your u3a and interest group. Check back each week for the solutions and let us know how you get on by contacting the u3a office. New maths puzzles will go up onto the website every Thursday.

**Question 1.**

Remove one digit from 2025 to make the largest possible number divisible by 9.

**Solution**

Divisibility by 9 requires the digit sum to be a multiple of 9.

The original digit sum is $2 + 0 + 2 + 5 = 9$ so, only the digit 0 can be removed to maintain a digit sum that is also a multiple of 9.

The largest valid number is 225.

*Alternative approach*

The solution can also be found by removing the smallest digit and testing if the remaining number is divisible by 9. Removing 0 gives 225 which is divisible by 9. There is then no need to look at removing any of the other digits

**Question 2.**

A metal cylinder, 12 cm diameter and 8 cm long, is to be melted down and recast as a sphere. What will be the diameter of the resulting sphere?

**Solution**

Suppose that the radius of the resulting sphere is r, then:

$\pi \times 6^2 \times 8 = \frac{4}{3} \times \pi \times r^3$

So, $r^3 = 6^2 \times 8 \times \frac{3}{4} = 6^3$

$r = 6$

The diameter of the sphere is 12 cm.

**Question 3.**

Find all prime numbers $p$ such that if $p^4 + 2$ is prime then so is $p + 4$.

**Solution**

Any prime number greater than 3 must be of the form $6k \pm 1$ for some positive integer $k$, because:

- a number of the form $6k$ is divisible by 6
- a number of the form $6k \pm 2$ is divisible by 2
- a number of the form $6k + 3$ is divisible by 3

If $p = 6k + 1$ is prime we can write:

$p^4 + 2$

$= (6k + 1)^4 + 2$

$= (6k)^4 + 4 \times (6k)^3 + 6 \times (6k)^2 + 4 \times 6k + 1 + 2$

$= 3 \times 2 \times 6^3k^4 + 3 \times 8 \times 6^2k^3 + 3 \times 2 \times (6k)^2 + 3 \times 8k + 3$

$= 3 \times (2 \times 6^3k^4 + 8 \times 6^2k^3 + 2 \times (6k)^2 + 8k + 1)$

So $p^4 + 2$ is not prime because it has a factor of 3 and $p^4 + 2$ cannot be equal to 3.

Similar logic shows that if $p = 6k - 1$, $p^4 + 2$ again cannot be prime.

This leaves the cases $p = 2$ and $p = 3$ to consider.

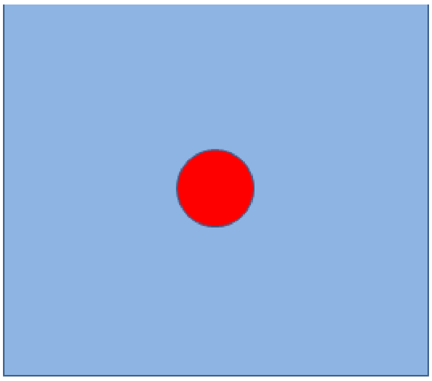
If $p = 2$, $p^4 + 2 = 18$ which is not prime.

If $p = 3$, $p^4 + 2 = 83$ which is prime, and $p + 4 = 7$ is also prime.

Thus, the question has just one solution, namely $p = 3$.

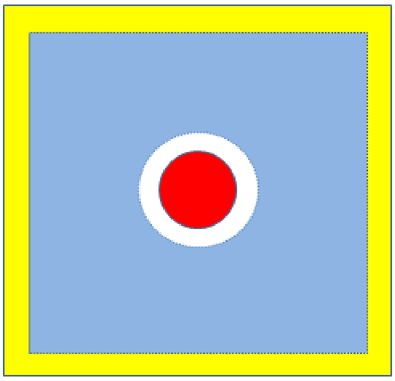
**Question 4.**

The diagram below shows the surface of a 50 cm by 50 cm square game board. The radius of the red circle is 5 cm and the centre of the circle coincides with the centre of the square. There is a raised edge around the perimeter of the square and when a circular disc of radius 2 cm is rolled on to the board, the raised edge means that all of the disc lands within the square. On the assumption that the disc is equally likely to land on any part of the board, what is the probability that no part of the disc covers any part of the red circle?



**Solution**

As the radius of the disc is 2 cm. the centre of the disc must be at least 2 cm from the edges of the square. The centre cannot therefore land in the yellow area in the diagram below, where the width of the yellow band is 2 cm. The centre of the disc can therefore land anywhere within a 46 cm by 46 cm square.



The white ring in the diagram has a width of 2 cm and if the centre of the disc lands in the white ring or red circle, part of the disc will overlap the red circle. The white ring and red circle form a circle of radius 7 cm so its area is $49\pi$ $cm^2$.

The probability that no part of the disk will overlap the red circle is therefore $(46^2 - 49\pi)/46^2 = 0.93$ to two decimal places.