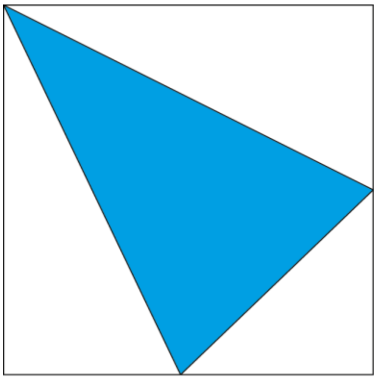
Maths Challenge - Week 292 – Solutions

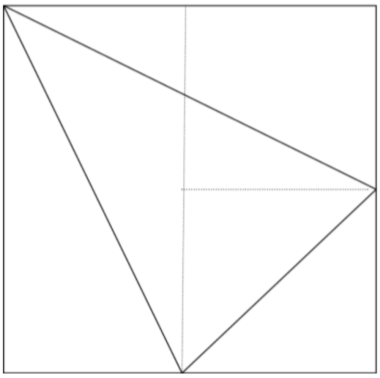
Welcome to week 292 of our weekly maths challenge, with problems and puzzles posed by David Browning, Rod Marshall, Ian Stewart, Annie Stothers and the u3a Maths and Stats Subject Adviser - David Martin. If you would like to share your ideas on how to solve these puzzles please join our learning forum or discuss within your u3a and interest group. Check back each week for the solutions and let us know how you get on by contacting the u3a office. New maths puzzles will go up onto the website every Thursday.

Question 1.



What fraction of the square has been shaded, given that the central shaded triangle has two of its vertices at the midpoints of sides of the square?

Solution



The dotted lines illustrate that the areas of the outer triangles are 1/4, 1/4 and 1/8 of the area of the square leaving 3/8 of the area of the square for the central, shaded, triangle.

Question 2.

Cate and her friend cycle from A to C, passing through B. Before they reach B, Cate asks her friend how far they have cycled. Her friend replies “one third as far as it is from here to B”. Ten miles later (after they have passed B) Cate asks her how far they must cycle to reach C. Her friend replies again “one third as far as it is from here to B”.

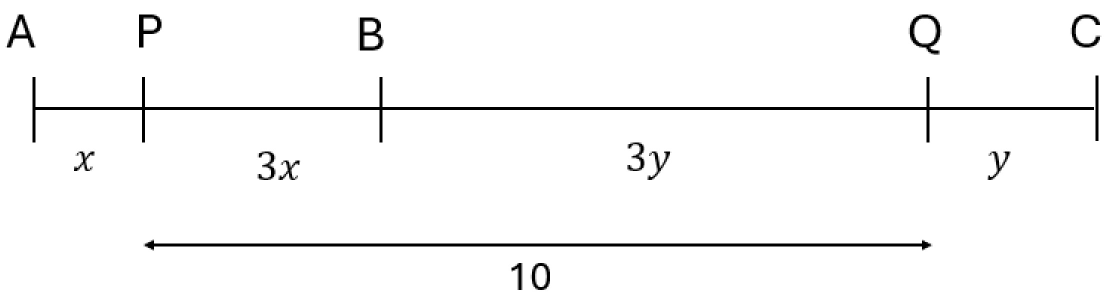
How far is it from A to C?

Solution

Let the points at which Cate asked each question be P and Q. Let the distances AP and QC be x miles and y miles respectively.

The distance from P to B is 3x miles and the point P must lie between A and B.

The distance from B to Q is 3y miles.



Since they have cycled 10 miles between P and Q, we know that 3x + 3y = 10

So, AC = 4x + 4y = 4(3x + 3y)/3 = 40/3 = 13 1/3 miles

Question 3.

Which positive integers are equal to 5 times the sum of their digits?

Solution

Any such integer must clearly have at least two digits, since no single digit number greater than zero can equal 5 times itself.

Suppose there is such an integer n with three digits. The sum of the digits of n cannot exceed 27, and 5 x 27 = 135. So, the first digit of n must be 1, which means the sum of the digits of n cannot exceed 19 (if n = 199). But 5 x 19 = 95 which has only two digits, so there are no integer solutions with three digits.

Similarly, integers with more than three digits can be ruled out. Consider an integer with d > 3 digits. The sum of the digits cannot exceed 9d, and 5 times this is 45d. For this to have at least four digits, d must be greater than 22 (as 45 x 22 = 990). But a 23-digit number is far, far greater than 23 x 9 x 5 = 1,035.

What about two-digit integers? Let t be the digit in the tens place and u be the digit in the units place. Then the number equals 10t + u and the sum of its digits is t + u. This means we need solutions to:

$10t + u = 5t + 5u \Rightarrow 5t = 4u$ and so, $t / u = 4 / 5$.

Since t and u are both single digits, t must be 4 and u must be 5, so the only answer to the question is 45.

Question 4.

Andrew, Beth, Charlie and Diana will receive several twenty-pound thank you vouchers from their boss Elaine. In how many ways could Elaine distribute her nine vouchers if each person receives at least one voucher?

Solution

As each person receives at least one voucher, four of the vouchers can be accounted for. One split of the other five vouchers would be to give Andrew, Beth, Charlie and Diana 1, 2, 0 and 2 vouchers; we could represent this split as V/VV//VV and this leads us to realise that the three / can be placed in $\binom{8}{3} = 56$ ways.