Week Fifty-Six Problems and Solutions

Question 1.

Consider all the three-digit numbers that can be formed using each of the numbers 1, 2 and 3 just once. What is their sum?

Solution

There are 6 such numbers. Each of 1, 2, 3 appears twice as a unit, a ten, and a hundred.

So, the sum is:

$2 \times (1 + 2 + 3) \times (100 + 10 + 1) = 12 \times 111 = 1332$

Question 2.

Three Mondays of a month fall on even numbered dates. Which day of the week was the 21st of the month?

Solution

If three Mondays of a month fall on even dates then there are five Mondays in the month**.** These must be 2nd, 9th, 16th, 23rd, and 30th. So, the 21st fell on a Saturday.

Question 3.

If a train heading east takes 4 seconds to pass a particular point while a train of equal length heading west takes 5 seconds, how long will it take the two trains to pass each other?

Solution

In one second, the eastbound train travels one-quarter of its length while the westbound train travels one-fifth of its length, so they separate by $^1/_4 + {}^1/_5 = {}^9/_{20}$ of the length of either train in one second. To pass each other, they must travel the combined length of the two trains, which will take $2 \times {}^{20}/_9 = 4{}^4/_9$ seconds.

Question 4.

At a stall at a village fete, there is a bag containing 16 table tennis balls. 2 of the balls have the number 10 on them; 3 have the number 5; 5 have the number 2; and 6 have the number 1. Anyone who pays £1 can choose two balls from the bag. The first ball chosen is not put back into the bag until after the second ball has been chosen. If the numbers match, their prize is the number of pounds shown on the ball e.g., £10 if they choose 2 balls with the number 10. If £200 is paid in entry fees, how much would the stallholder expect (on average) to pay in prizes?

Solution

The probability of the first ball having a 10 on it is 2/16 and the probability of the second ball having 10 on it is 1/15. The probability of winning £10 is 2/16 x 1/15.

Similarly, the probability of winning £5 is 3/16 x 2/15; probability of winning £2 is 5/16 x 4/15; and the probability of winning £1 is 6/16 x 5/15.

The expected (on average) prize money is therefore:

200 x [(£10 x 2) + (£5 x 6) + (£2 x 20) + (£1 x 30)] / (16 x 15)

= 200 x £120 / (16 x 15) = £100