Problems and solutions

Week Fifty-five

Question 1.

Five chocolate cakes and five jam tarts are worth the same as two chocolate cakes and eleven jam tarts. How many jam tarts is a chocolate cake worth?

Solution

Suppose the value of a chocolate cake is c and the value of a jam tart is t.

Then 5c + 5t = 2c + 11t and hence 3c = 6t and c = 2t. So, a chocolate cake is worth two jam tarts.

Question 2.

Two types of tickets were sold for a concert held at an amphitheatre. Tickets to sit on a bench during the concert cost £75 each, and tickets to sit on the lawn during the concert cost £40 each. Organizers of the concert announced that 350 tickets had been sold and that £19,250 had been raised through ticket sales alone. What number of bench seats and lawn seats were sold?

Solution

If the number of bench seats sold was B and the number of lawn seats sold was L.

Then, $B + L = 350$ or $L = 350 - B$
And $75B + 40L = 19{,}250$

So, $75B + 40(350 – B) = 19{,}250$

$75B + 14{,}000 – 40B = 19{,}250$

$35B = 5{,}250$

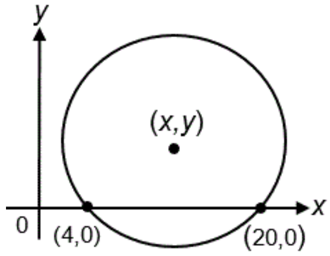
$B = 5{,}250 / 35 = 150$

$L = 350 – B = 350 – 150 = 200$

So, 150 bench seats and 200 lawn seats were sold.

Question 3.

In the *xy*-plane below, the circle has centre (*x,y*) and a radius of 10. What are the value for the centre coordinates (*x,y*)?



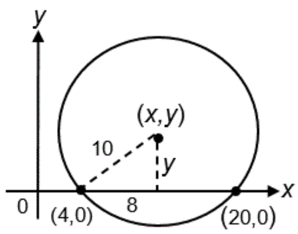
Solution

$x = 4 + (20 – 4)/2 = 12$ and

$y^2 = 10^2 - 8^2 = 100 – 64 = 36$

$y = 6$

Therefore, circle centre (*x,y*) is at (12,6)



Question 4.

Numbers are said to be in arithmetic progression if the difference between successive numbers is constant – e.g. 4, 7, 10, 13 …

Find three integers in arithmetic progression whose product is prime.

Solution

As a prime number has no factors other than 1 and itself (by definition), you may think there can be no solution to this. But the question refers to integers, not whole numbers, so we are able to include negative integers in the arithmetic progression.

If there is a solution involving negative integers, then as prime numbers are all positive (again by definition), precisely two of the three integers in the progression must be negative.

Suppose the three terms of the progression are a - d, a, a + d. Then two of $(a – d)^2$, a^2, $(a + d)^2$ must equal 1, otherwise the product of the three terms in the progression could not be prime.

If both $(a - d)^2 = 1$ and $(a + d)^2 =1$ then by subtraction we get 2ad = 0. But d cannot be zero, so a would be 0 which would give a product of terms of 0 which is not prime.

So $a^2 = 1$ which means a = ±1. But as a is the middle term in the progression, it must be negative (if a were positive, the other two terms could not both be negative), so a = -1.

We now have that *either*

$(a - d)^2 = a^2 – 2ad + d^2 = 1 – 2ad + d^2 = 1$ so $d(d – 2a) = 0$ and $d = 2a = -2$

or

$(a + d)^2 = a^2 + 2ad + d^2 = 1 + 2ad + d^2 = 1$ so $d(2a + d) = 0$ and $d = -2a = 2$

So, there are two progressions which satisfy the question, one being the reverse of the other:

-3, -1, 1 and 1, -1, -3