Problems and solutions

Week Fifty-two

Question 1.

Washington High School randomly selected freshmen, sophomore, junior, and senior students for a survey about potential changes to next year's schedule. Of students selected for the survey, 1/4 were freshmen and 1/3 were sophomores. Half of the remaining selected students were juniors. If 336 students were selected for the survey, how many were seniors? (For clarity, a student is either a freshman, sophomore, junior or senior student.)

Solution

1/4 + 1/3 = 7/12 of the students were freshmen or sophomores, leaving 5/12 = 10/24 of the students of which half, 5/24, were juniors and the other half, 5/24, were seniors. The number of seniors was therefore 336*5/24 = 70

Question 2.

When asked about the animals on his farm, the farmer replied: "I only keep sheep, cows and horses. At the moment, all bar 20 are sheep, all bar 30 are cows and all bar 40 are horses."

How many of each animal are on the farm?

Solution

Adding the numbers 20, 30 and 40 will count every animal twice.

So, there are (20 + 30 + 40) / 2 = 45 animals in total on the farm, of which:

45 – 20 = 25 are sheep

45 – 30 = 15 are cows

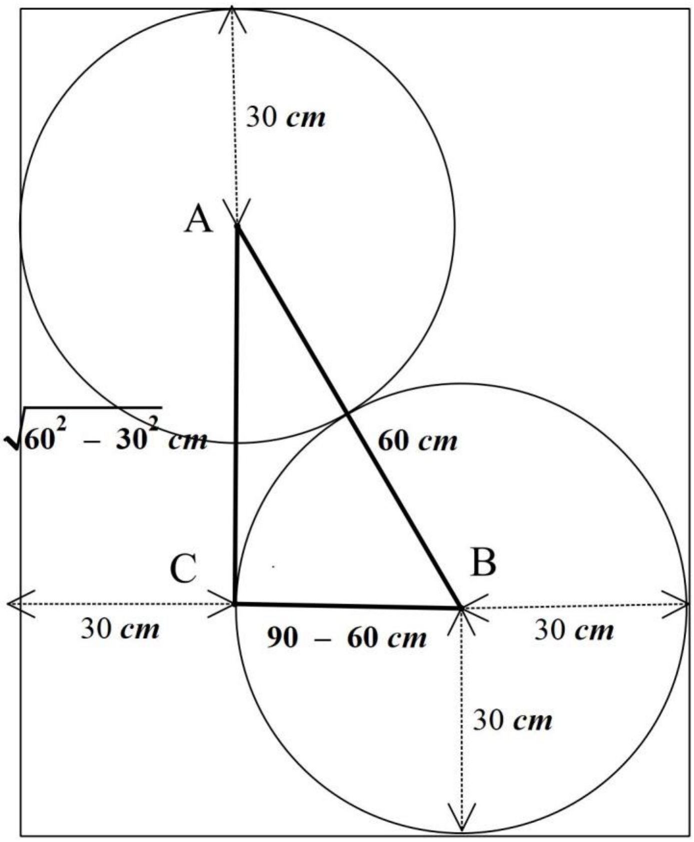
45 – 40 = 5 are horses

Question 3.

To make a circular cushion, a member of a U3A sewing group needs two full circles of fabric each with a diameter of 60 cm. If the fabric they want to use is 90 cm wide, what is the minimum length of fabric that they have to buy?

Solution

As the diameter of the circles is more than half the width of the fabric, the two circles have to be offset. The minimum length of fabric will be required if one circle is touching two adjoining sides of the fabric, the other circle is touching the other two adjoining sides and the two circles are touching. This is shown in the following diagram. The rectangle represents the fabric with the vertical representing the length and the horizontal the width of 90 cm.



In the right-angled triangle ABC, BC = 90 – 2x30 = 30 cm and AB = 2x30 = 60 cm, hence, by Pythagoras' theorem AC = $\sqrt{60^2 - 30^2} = 30\sqrt{3}$ cm. The height of the rectangle, and therefore the minimum length of fabric required, is $2 \times 30 + 30\sqrt{3} = 30(2 + \sqrt{3}) = 112\ cm$.

Question 4.

Find the solutions to the simultaneous equations:

$3x^2 + xy - 2y^2 = -5$

$x^2 + 2xy + y^2 = 1$

Solution

From the second equation: $(x + y)^2 = 1$

So, $x + y = \pm 1$ and therefore $y = (1 - x)$ or $y = -(1 + x)$

Inserting $(1 - x)$ for y in the first simultaneous equation gives:

$3x^2 + x(1 - x) - 2(1 - x)^2 = -5$

$(3 - 1 - 2)x^2 + (1 + 4)x + 3 = 0$ so $x = -3/5$ and $y = 8/5$

Inserting $-(1 + x)$ for y in the first simultaneous equation gives:

$3x^2 – x(1 + x) - 2(1 + x)^2 = -5$

$(3 – 1 – 2)x^2 – (1 + 4)x + 3 = 0$ so $x = 3/5$ and $y = -8/5$