Week Fifty-three Problems and Solutions

Question 1.

The price of a toy was marked down by 40% in a sale. If the sale price was £18, what was the original price of the toy?

Solution

Let p = original price of toy
Then, p x 0.60 = £18
So, the original price of the toy was p = £18 / 0.6 = £30

Question 2.

In 2015 the populations of two cities were equal. From 2010 to 2015, the population of the first of these cities increased from 120,000 by 20% and the population of the second city decreased by 10%. What was the population of the second city in 2010?

Solution

Let s = population of the second city in 2010
Then 120000 x 1.20 = s x 0.9
So, the population of the second city in 2010 was s = 120000 x 1.20 / 0.9 = 160,000

Question 3.

A company sells its products in cylindrical containers which are 8 cm high and have a circular base with radius π cm. The product is delivered to shops in cardboard boxes holding 12 containers, arranged in 4 rows of 3. To reduce cardboard usage, the company decides to pack its products in containers which are cubes each holding the same amount of product as the current cylindrical containers. By what percentage will the company reduce the amount of cardboard used for the top and bottom of a box of 12 containers; and what is the percentage reduction for cardboard used for the sides?

Solution

The dimensions of the cardboard box for the 12 cylindrical containers are 8π x 6π x 8. The area of each of the top and bottom is 8π x 6π cm^2.

The volume of the cylindrical container is $8\pi^3$ cm^3. This volume can be held in a cube of side 2π cm. The dimensions of a cardboard box to hold 12 cube containers of this size will be 8π x 6π x 2π and the area of the top and bottom is 8π x 6π cm^2. There is therefore no reduction in cardboard used for top and bottom.

The surface area of the sides of the box is the height of the box multiplied by the perimeter of the top. The perimeter is the same for both boxes. So, the ratio of the surface area of the sides will be the ratio of the heights. This ratio is $2\pi/8$ ie $\pi/4 = 0.785$. The percentage reduction in cardboard usage for the sides is 21.5%.

Question 4.

Tom, Dick and Harry are brothers and their ages are all prime numbers. The difference between the ages of any two of them is also prime. What are the three ages?

Solution

0 is not a prime, hence. the boys ages are different, and as only one prime is even at least two of them must be odd.

If all three prime numbers are odd then the difference between the smallest and largest of them will be an even number greater than 2 and hence not prime.

So, one of the boys is of age 2.

The two larger numbers are odd primes, so the difference between them is even and prime.

So, it is 2. The numbers can therefore be written as 2, x, x + 2

The differences between the numbers are prime. That means x - 2 is a prime as well as x and x + 2.

Suppose (x - 2) is a prime greater than 3, then it must be 1 away from a multiple of 6

If $x – 2 = 6n -1$ then $x + 2 = 6n + 3$ and is divisible by 3

If $x – 2 = 6n +1$ then $x = 6n + 3$ and is divisible by 3

So, if all three are prime, it follows that x – 2 cannot be a prime greater than 3

Hence $x – 2 = 3$; $x = 5$; and the three ages are 2, 5, and 7