Week Fifty-four Problems and Solutions

Question 1.

A cargo helicopter delivers only 50 and 60 kg packages. For each delivery trip, the helicopter must carry at least 10 packages, and the total weight of the packages cannot exceed 550 kg. What is the maximum number of 60 kg packages that the helicopter can carry per trip?

Solution

At maximum load the helicopter can carry 11 of the lighter 50 kg packages. Removing one of these 50 kg packages to reduce the package count down to the minimum of 10 packages would enable the helicopter to carry the maximum of 5 heavier 60 kg packages.

Question 2.

Mary‘s mother has twice as many sisters as brothers. Each sister has 3 children, and each brother has 4 children. Mary’s father has twice as many brothers as sisters. Each sister has 3 children, and each brother has 2 children. If Mary has 27 cousins, how many siblings does her mother have?

Solution

If Mary’s mother has B brothers, she has 2B sisters. The number of children which Mary’s mother’s siblings have is therefore (3x2B) + 4B = 10B. If Mary’s father has S sisters, he has 2S brothers. The number of children which Mary’s father’s siblings has is 3S + (2x2S) = 7S.

As Mary has 27 cousins, 10B + 7S = 27. As B and S must be whole numbers, B = 2 and S = 1. Mary’s mother has 2 brothers and 4 sisters so 6 siblings in total.

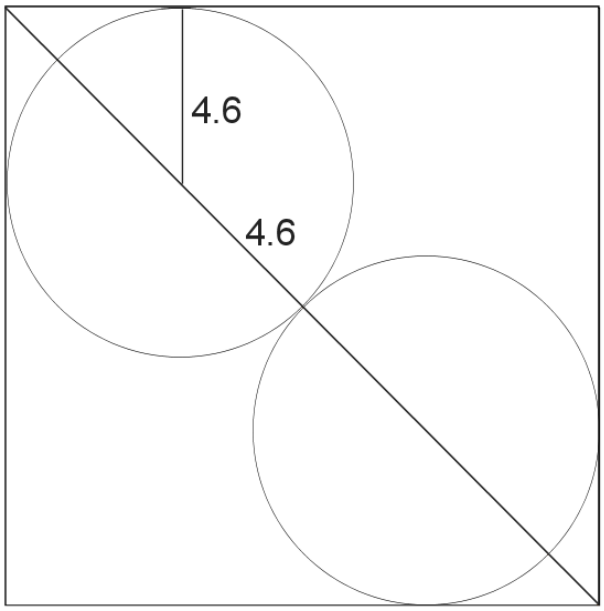
Question 3.

Croquet balls come in sets of four, and the ones in my set have a diameter of 9.2cm. What is the size of the smallest cubical box that could hold the set of balls?

(The diameter of a regulation-sized croquet ball is $3^5/_8$ inches, but such a number is awkward when it comes to doing calculations. $3^5/_8$” ≈ 9.2cm.)

Solution

To fit into the smallest possible cube, two of the balls must be placed along a diagonal of the bottom of the box, then the other two balls placed on top of the first two along the other diagonal; the centres of a close cluster of four balls form the vertices of a tetrahedron.



Looking down into the cube from above at just the two balls in the lower layer (see diagram) and utilising Pythagoras, we can see that the length of the diagonal is

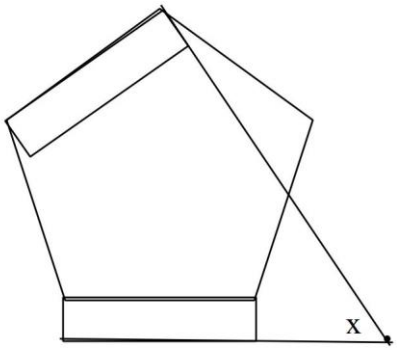
$2 \times [4.6 + \sqrt{(2 \times 4.6^2)}] \approx 22.21$ cm.

The edges of the cubical box therefore have a length of

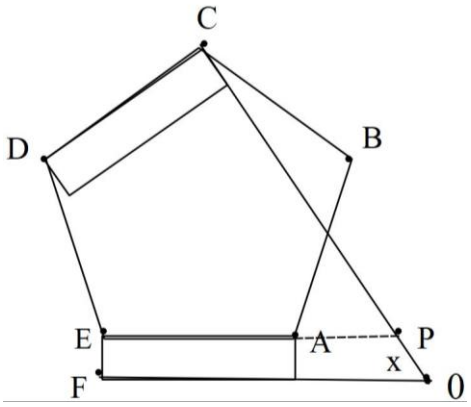
$\sqrt{(22.21^2 / 2)} \approx 15.7$ cm.

Question 4.

Two rectangles are drawn on the sides of a regular pentagon as shown in the diagram below and two of their sides are extended. What is the angle x?



Solution



Extend EA to meet CO at P.

Angles CDE and DEA are 108° since ABCDE is a regular pentagon.

Then, using the angles of quadrilateral PCDE

x = Angle EPC (as EP is parallel to FO) = 360 – 90 - 2 x 108 = 54°