Problems and solutions

Week Forty-seven

Question 1.

Mr Kohl has a beaker of solution to distribute to the students in his chemistry class. If he gives each student 3 millilitres (ml) of solution, he will have 5 ml left over. In order to give each student 4 ml of solution, he will need an additional 21 ml. How many students are in the class?

Solution

Let s be the number of students in Mr Kohl's class.
And q be the quantity of solution to be distributed.
Then, $q = (3 * s) + 5$ and $q + 21 = (4 * s)$
Substituting for q, then $(3s + 5) + 21 = 4s$,
So, $3s + 26 = 4s$
Therefore, the number of students, $s = 26$

Question 2.

The sum of three numbers is 855. One of the numbers is 50% more than the sum of the other two numbers. What is the value of this largest number?

Solution

Let the largest number be x and the sum of the other two numbers be y.
Then, $x + y = 855$ and $x = 3y/2$, so, $y = 2x/3$
$x + y = x + 2x/3 = 5x/3 = 855$
So, $x = 513$

Question 3.

Find four whole numbers so that any number from 1 to 40 can be formed by adding and subtracting a selection of them.
For example, with 1, 2, 5 and 10 we can form $14 = 10 + 5 – 1$,
However, the highest number possible with these four numbers is $10 + 5 + 2 + 1 = 18$

Solution

Each of the four numbers can be added, omitted, or subtracted in a sum, to form at most $3x3x3x3 = 81$ possible results. One result will be zero and each positive result will have a corresponding negative result (obtained by swapping additions and subtractions). Only if the four numbers are different can we create 40 different positive results.
Consider the four numbers in increasing size. If we start with 1, then if the second number is n, then additionally $n – 1$ to $n + 1$ can be formed, and so we require $n = 3$ to

be able to additionally form 2 to 4. If the third number is m, then additionally m – 4 to m + 4 can be formed, and so we require m = 9 to be able to additionally form 5 to 13. The fourth number can likewise be determined to be 27 or can be deduced from the fact that all the four numbers will add up to 40. With 1, 3, 9 and 27 we can form 1 to 40. You may wish to consider if there are any other solutions.
What might be possible with five numbers?

Question 4.

Adam, Mary and Tom each have a bag of apples. Mary has 50% more than Adam and Tom has 50% more than Mary. Tom gives 6 apples to Adam. The three all eat one of their apples. Each of the three now has a two-digit prime number of apples. How many apples did Adam have at the start?

Solution

Suppose that A, M and T are the number of apples that Adam, Mary and Tom have at the start.
M = 1.5 x A and T = 1.5 x M so T = 1.5 x 1.5 x A = 2.25 x A
A must be divisible by 4 so that Tom has a whole number of apples.
So, we can write A, M and T as A = 4 x N, M = 6 x N, T = 9 x N, for some whole number N
Adam has received 6 apples from Tom and has eaten 1 so Adam now has (4N + 5) apples
Mary has eaten 1 apple so she now has (6N – 1) apples
Tom has given 6 apples to Adam and has eaten 1 so he now has (9N – 7)
11, 29, 47 and 83 are the only two-digit primes that can be derived from (9N – 7) and these correspond to N = 2, 4, 6 and 10.
Putting those values of N into (6N – 1) gives 11, 23, 35 and 59. 35 is not prime so N cannot be 6
Putting the remaining 3 possible values of N into (4N + 5) gives 13, 21, 45.
13 is the only prime and that corresponds to N = 2
As Adam had 4N apples, he had 8 apples.