Problems and solutions

Week Forty-eight

Question 1.

Josh and Ravinder both collect slide rules. Josh has three times as many slide rules in his collection as Ravinder. If Josh gave three of his collection to Ravinder then his collection would be twice the size of Ravinder's. How many slide rules does each have?

Solution

Suppose that Josh has J slide rules and Ravinder has R slide rules.
Then, $J = 3R$ and $J – 3 = 2 (R + 3)$
So, substituting for J in the second equation, $3R – 3 = 2R + 6$
Hence, $R = 9$, and $J = 3R = 27$
Ravinder has 9 slide rules and Josh has 27 slide rules.

Question 2.

Two large and 1 small pump can fill a swimming pool in 4 hours. One large and 3 small pumps can also fill the same swimming pool in 4 hours. How many hours will it take 4 large and 4 small pumps to fill the swimming pool?

Solution

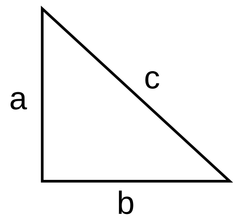
By comparing the first two sentences of the question, we see that substituting one large pump with two small ones makes no difference to the time taken to fill the pool. Two small pumps are therefore equivalent to one large pump, and so from the second sentence we know 5 small pumps will fill the pool in 4 hours.

Four large and four small pumps are equivalent to $4 \times 2 + 4 = 12$ small pumps, so these will take 5/12ths of 4 hours to fill the pool, or 1h 40m.

Question 3.

Find all sides of a right-angled triangle whose perimeter is 60 cm and whose area is 150 cm^2.

Solution



$a + b + c = 60$
$½ (a * b) = 150$
So, $a + b = 60 – c$ and $ab = 300$
By Pythagoras' theorem, $c^2 = a^2 + b^2$
Then, $c^2 = a^2 + b^2 = (a + b)^2 - 2ab = (60 - c)^2 - (2 * 300)$
$(3600 – 120c + c^2) - 600 = c^2$
$120c = 3000$ and so, $c = 25$ cm
Then, $a + b = 60 – c = 60 – 25 = 35$
So, $b = 35 - a$
Given, $a * b = 300$, then, $a(35 – a) = 300$
$a^2 - 35a + 300 = 0$ or $(a - 15) * (a - 20) = 0$
Therefore, $a = 15$ and $b = 20$ or $a = 20$ and $b = 15$
That is, the triangle has sides of length 15 cm, 20 cm, and 25 cm.

Question 4.

You had three identical crystals that will shatter if dropped from a certain height or above onto a pavement. The crystals will be undamaged if dropped from a lower height.

One crystal has already been dropped from the 11th floor of a building and it shattered on hitting the pavement below. You now want to establish the lowest floor from which a crystal will shatter if it is dropped. How do you keep the maximum number of drops you may have to make to a minimum?

Solution

Drop the 2nd crystal from the 4th floor
If it breaks, you have to drop the 3rd crystal from the 1st floor, and keep moving up a floor until it breaks (If it doesn't break, the fourth floor is the one you're looking for.)
Otherwise, drop the 2nd crystal from the 7th floor
If it breaks, drop the 3rd crystal from the 5th and if it hasn't broken, the 6th floor
Otherwise, drop the 2nd crystal from the 9th floor
If it breaks, drop the 3rd crystal from the 8th floor
Otherwise, drop the 2nd crystal from the 10th floor
The above method will require at most 4 drops

Why start from the 4th floor?

Suppose your first drop is from floor F ($1 \le F \le 10$).

If the crystal shatters, you will then have no option but to drop the final crystal from floor 1 and work upwards floor by floor until that crystal shatters. This could involve a total of F drops.

If the crystal does not shatter when dropped from floor F, move up (F - 1) floors and make your second drop from floor 2F - 1. If the crystal shatters, then you have to drop the final one from floor F + 1 and work up floor by floor as before. Again, you may need F drops in total, but no more than that.

If the crystal does not break when dropped from floor 2F - 1, continue advancing up the building by one less floor than before, i.e. by (F - 2) floors then (F - 3) floors and so on until the crystal shatters (and then repeat the rest of the process as before) or until you reach the top of the building.

The value of F which meets the condition in the question is the lowest number such that $F + (F - 1) + (F - 2) + ... + 1 \geq 10$.
The sum on the left-hand side gives the so-called triangular number $T_F = F \times (F + 1)/2$.
We need the lowest F such that $F \times (F + 1) \geq 20$, from which $F = 4$.

What if the first crystal had been dropped from, say, the 101st floor?