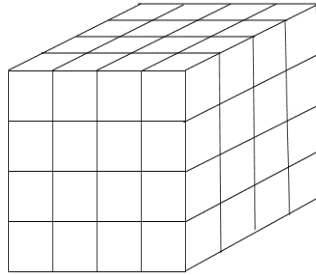


Week Forty-two problems and solutions

Question 1.

The cube shown is made of a small cubes of sides 2 cm. When the large cube is disassembled into its component small cubes, the total surface area of all the small cubes is how many times greater than the surface area of the large cube?



Solution

The surface area of each small cube = $6 \times 2 \times 2$

The surface area of the large cube = $6 \times 8 \times 8$

Therefore, the total surface area of the 64 small cubes is $(64 \times 6 \times 2 \times 2) / (6 \times 8 \times 8)$
= 4 times the surface area of the large cube.

Question 2.

The square of a positive real number is twice the size of the cube of that number. What is the number?

Solution

Let the number be x . Then $x^2 = 2x^3$ and $x^2(1 - 2x) = 0$.

So, $x = 1/2$ as $x = 0$ is not a positive number.

Question 3.

Zoe began gardening at 10:20am. She spent twice as much time clipping the hedge as mowing the lawn and ten minutes more time weeding than hedge clipping. She finished at ten minutes after noon. How long did Zoe spend on each task?

Solution

Suppose that the times, in minutes, spent on hedge clipping, lawn mowing and weeding were c , m and w

Then, $c = 2m$, $w = c + 10$, $c + m + w = 110$

So, $c + m + w = c + m + (c + 10) = 2c + m + 10 = 2(2m) + m + 10 = 110$

$5m + 10 = 110$

$m = 20$, hence $c = 2 \times 20 = 40$ and $w = 40 + 10 = 50$

Thus, 20 minutes were spent on mowing, 40 minutes on hedge clipping and 50 minutes on weeding.

Question 4.

Show that for any ten consecutive positive integers at least one is coprime (no common factors other than 1) to the product of the others.

Solution

Five of the numbers will be odd

At most two of these odd numbers will be divisible by 3

At most one of these odd numbers will be divisible by 5

At most one of these odd numbers will be divisible by 7

So, at least one of these odd numbers will not be divisible by 2, 3, 5 or 7.

Hence, there is a number with all its prime factors greater than 10.

As all the other numbers are less than 10 apart from this number none of them will be divisible by any of these prime factors.

Hence, this number will be coprime to all the others and therefore coprime with their product.