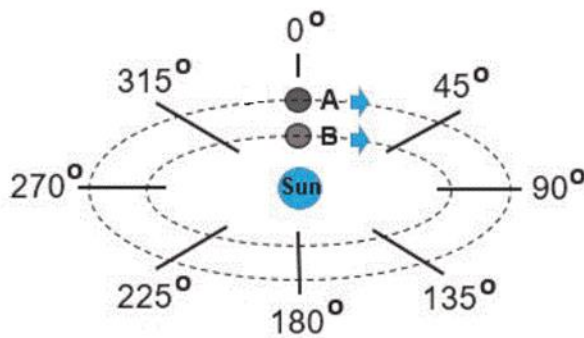


Week Forty-three solutions

Question 1.

Two planets are in line with each other and the sun. The outer planet A will orbit the sun every twelve years. The inner planet B takes three years. Both move in a clockwise direction. When will they next form a straight line with each other and the sun?



Solution

Planet A: $360^\circ / 12 = 30^\circ/\text{yr.}$ and Planet B: $360^\circ / 3 = 120^\circ/\text{yr.}$

Let x = numbers of years to be in line again with the sun.

$120x - 30x = 180^\circ$, $90x = 180^\circ$, therefore, $x = 2$ years

Planet A at 60° and planet B at 240° in their orbit.

Question 2.

A circular logo is enlarged to fit the lid of a jar. The new diameter is 50 per cent larger than the original. By what percentage has the area of the logo increased?

Solution

If the diameter of old lid is $4d$, then the diameter of the new lid will be $6d$. The areas of the lids will be in the ratio of $\pi(2d)^2 : \pi(3d)^2$ or $4 : 9$. This is an increase in area of $5/4$ or 125%.

Question 3.

A laboratory has home and visiting researchers. Each home researcher has two more home colleagues than visiting colleagues and each visiting colleague has half as many visiting colleagues as home colleagues. How many home and visiting researchers work in the laboratory?

Solution

Let H = number of home researchers and V = number of visiting researchers

$$H - 1 = V + 2 \text{ and } V - 1 = \frac{1}{2} H,$$

$$H = V + 3 \text{ and } H = 2V - 2$$

$$\text{Then } V + 3 = 2V - 2, 3 + 2 = 2V - V = V, V = 5$$

$$\text{And } H = V + 3 = 5 + 3 = 8$$

The number of visiting researchers is 5 and the number of home researchers is 8.

Question 4.

Find the sum of the first 75 terms of the arithmetical progression: 60, 40, 20,

...

Solution

Dividing each term by 20 consider the progression 3, 2, 1, ... with 75th term (-71)

Suppose that the sum of these is S

$$S = 3 + 2 + 1 + \dots + (-69) + (-70) + (-71)$$

Reversing the terms

$$S = (-71) + (-70) + (-69) \dots + 1 + 2 + 3$$

$$\text{Adding in pairs, } 2S = -68 \times 75 \text{ i.e. } S = -68 \times 75 / 2 = -2550$$

The sum of the original progression is therefore $-2550 \times 20 = -51,000$