Week Forty-one problems and solutions

Question 1.

In my sock drawer I have 10 red socks, 12 black socks and 8 blue socks, all loose and unpaired. I am off on a business trip but, because of a power cut, I am packing in the dark. How many socks must I pack to ensure I have a pair; and how many to ensure I have a black pair; and what is the chance of getting a red pair, if I pack only two socks?

Solution
For a pair, with three socks there may be one of each colour so 4 socks will be needed to guarantee that I have a pair.
For a black pair, I may have up to 10 red and 8 blue socks, so 20 socks will be needed to guarantee a black pair.
The chances that the first sock is red = 10/30 = 1/3 and the chance of the second sock being red = 9/29 and so the chance of getting a pair of red socks is 1/3 x 9/29 = 9/87 (around 10%).

Question 2.

My garden has two flower beds and three large pots, and this morning I took a tray of seedlings from my greenhouse to plant out. I put a third of them in the first bed and six in the first pot; I put a half of what was left in the second bed and 60% of the remainder in the second pot. This left me with just four for the last pot. How many seedlings did I start with?

Solution

Let N = total number of seedlings that I started with.

The number left after the first bed and pot = 2N/3 – 6

The number left after the second bed and pot = [(2N/3 – 6)/2] x 0.40 = 4

So, N/3 - 3 = 10

The number of seedlings at the start, N = 39

Question 3.

I toss a coin 6 times. The score for heads is 3 and for tails is 1. What is the probability that I score 12 in total?

Solution

The total number of possible scores is 2^6 = 64

The total score for n heads, and hence 6 − n tails, is $3n + (6 − n) = 2n + 6$
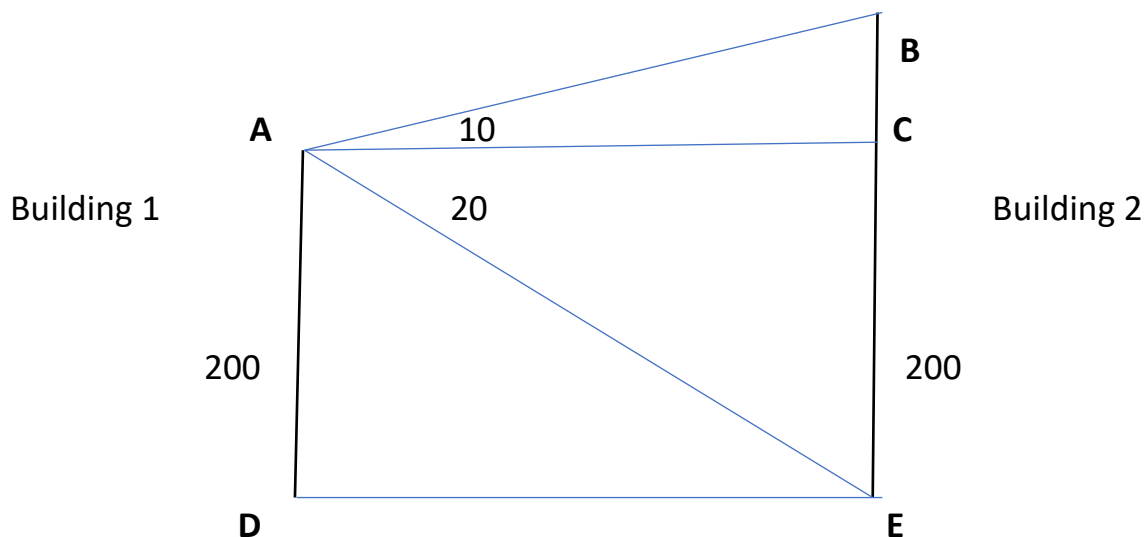
So, for a total score of 12, the number of heads, n = 3

These can occur in any order. So there are $^6C_3 = (6 \times 5 \times 4)/(3 \times 2 \times 1) = 20$ times this could occur.

Probability is 20/64 = 5/16.

Question 4.

From the top of a 200 meters high building, the angle of depression to the bottom of a second building is 20 degrees. From the same point, the angle of elevation to the top of the second building is 10 degrees. Calculate the height of the second building.

Solution

From triangle ACE tan 20 = 200/AC and hence AC = 200/tan 20

In triangle ABC tan 10 = BC/AC

So BC = AC tan 10 = 200 tan 10 / tan 20

So height of second building is 200 + 200 tan 10/tan 20 ≈ 200 + 97 = 297m