Problems and Solutions

Week Thirty-seven

Question 1.

An online bookstore sells novels and magazines. Each novel sells for £4, and each magazine sells for £1. If Fiona purchased a total of 11 novels and magazines that have a combined selling price of £20, how many novels did she purchase?

Solution

Let N = the number of novels and M = the number of magazines purchased.

4N + M = 20 i)
N + M = 11   ii)
Subtracting ii) from i)
3N = 9 i.e. N = 3
Fiona purchased 3 novels.

Question 2.

The mean score of 8 players in a basketball game was 14.5 points. If the highest individual score is removed, the mean score of the remaining 7 players becomes 12 points. What was the highest score?

Solution

The total score for 8 players was 8 x 14.5 =116
The total score for 7 players was 7 x 12 = 84
Therefore, the highest score = 116 – 84 = 32

Question 3.

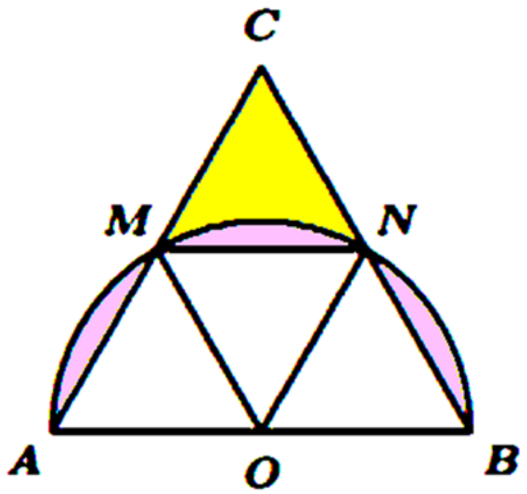
What is the smallest positive integer value of n for which $(2^2 – 1)(3^2 – 1)$ …….. $(n^2 -1)$ is a perfect square?

Solution

$(n^2 – 1) = (n - 1)(n + 1)$
Therefore, $(2^2 – 1)(3^2 – 1)$ …….. $(n^2 - 1)$
= (2 – 1)(2 + 1)(3 - 1)(3 + 1)…(n – 1)(n + 1)
= (2 - 1)(3 – 1)…(n – 1)(2 + 1)(3 + 1)…(n + 1)
= 1.2…(n - 1).3.4…(n + 1)
= [1.2.3…(n – 1)]^2 x n(n + 1)/2
We therefore require the smallest positive integer n for which the triangular number n(n + 1)/2 is also a square number.
The triangular numbers are 1, 3, 6, 10, 15, 21, 28, **36**,…
The smallest n that satisfies the condition is 8

Question 4.

A semicircle has diameter AB. The equilateral triangle ABC is drawn on the same side of AB as the semicircle.



Determine the area that lies inside the triangle CMN and outside the semi-circle.

Solution

Suppose the radius of the semi-circle is r.
Then the area of the semi-circle is $\pi r^2/2$.
Triangles AOM, OMN, ONB, and CMN are all equilateral with side r.
The area of each triangle is $\sqrt{3}r^2/4$
The area inside $\triangle$ABC but outside the semicircle is equal to the area of rhombus MONC minus the area of sector MON.
Area of rhombus MONC = $2 \times \sqrt{3}r^2/4 = \sqrt{3}r^2/2$
Area of circle sector MON is $\pi r^2/6$
So, area inside triangle CMN but outside the semicircle is $\sqrt{3}r^2/2 - \pi r^2/6$
$= (\sqrt{3}/2 - \pi/6)r^2$