## **Problems and Solutions**

## **Week Forty**

### Question 1.

Craig ran around a track, initially taking two minutes for each lap, then slowing down to allow himself three minutes for each lap. If he completed 24 laps in 58 minutes, how many laps did he run at the faster speed?

### Solution

Let f = number of faster laps and s = number of slower laps.

Then, f + s = 24

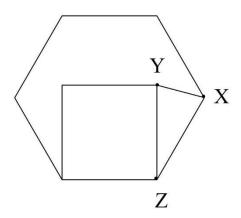
and, 2f + 3s = 58

Taking the second equation from 3 lots of the first equation gives f = 14

So, 14 laps were run at the faster speed.

## Question 2.

In the diagram below a square is set in a regular hexagon. What is the size of the angle YXZ?



# Solution

The interior angle of a regular hexagon is  $120^{\circ}$ . So  $\angle$  XZY =  $120 - 90 = 30^{\circ}$ .

The side of the square has the same length as the side of the hexagon. So, triangle XYZ is isosceles. Hence  $\angle$  YXZ =  $\frac{1}{2}$  (180 - 30) = 75°

### Question 3.

In 2 years', Bill's age will be three times the age of his son, John, was two years ago. Three years ago, Bill's age was twice the age John will be in three years' time. How old are they?

### Solution

Let b = age of Bill and j = age of John.

Then, 
$$b + 2 = 3(j - 2)$$
 i.e.  $b = 3j - 8$ 

and 
$$b - 3 = 2(j + 3)$$
 i.e.  $b = 2j + 9$ 

So, 
$$3j - 8 = 2j + 9$$
 i.e.  $j = 17$  and  $b = 2j + 9 = 43$ 

John is 17 and Bill is 43

## Question 4.

Items of fruit are sold singularly in a supermarket. From the information below can you list the seven fruits in order of price, starting with the most expensive?

A banana costs less than a peach but more than a pear. A kiwi costs less than a pear and more than an apple. A pineapple is more expensive than a peach. A lemon costs more than a kiwi but less than a pear.

### Solution

A banana (A) costs less than a peach (B) but more than a pear (C).

So, B > A > C

A kiwi (D) costs less than a pear (C) and more than an apple (E).

So, C > D > E

Hence, B > A > C > D > E

A pineapple (F) is more expensive than a peach (B).

So, F > B > A > C > D > E

A lemon (G) costs more than a kiwi (D) but less than a pear (C).

So, C > G > D

Hence, F > B > A > C > G > D > E

Thus, the order of fruits from more expensive to less expensive is

 $\mathsf{F}-\mathsf{pineapple},\,\mathsf{B}-\mathsf{peach},\,\mathsf{A}-\mathsf{banana},\,\mathsf{C}-\mathsf{pear},\,\mathsf{G}-\mathsf{lemon},\,\mathsf{D}-\mathsf{kiwi},\,\mathsf{E}\,\mathsf{-apple}$