## Problems and Solutions

## Week Thirty-four

### Question 1.

A group of friends decided to divide the £800 cost of a trip equally among themselves. When two of the friends decided not to go on the trip, those remaining still divided the £800 cost equally, but each friend's share of the cost increased by £20. How many friends were in the group originally?

### Solution

Let $n$ be the original group size.
Then the original cost per person = 800/n
With the smaller group size of n-2, the increased cost per person increased by £20 to 800/n + 20 = 800/(n – 2)
Multiplying each side by n(n - 2), gives

$800(n - 2) + 20n(n - 2) = 800n$
$800n - 1600 + 20n^2 - 40n = 800n$
$20n^2 - 40n - 1600 = 0$
$n^2 - 2n - 80 = 0$
$(n+8)(n-10) = 0$

The solutions of this equation are −8 and 10. Since a negative solution makes no sense for the number of people in a group, the number n of friends originally in the group was 10.

### Question 2

Jaime is preparing for a cycle race. His goal is to cycle an average of at least 280 miles per week for four weeks. He cycled 240 miles the first week, 310 miles the second week, and 320 miles the third week. How miles must Jaime cycle in the fourth week to meet his goal?

### Solution

Jaime's goal is to cycle a total of at least 4 x 280 = 1120 miles.
By the end of the third week he has cycled 240 + 310 + 320 = 870 miles
He will therefore need to cycle at least 1120 – 870 = 250 miles in the fourth week.

**Question 3.**

How many integers $\alpha$ are there such that the roots of $x^2 + \alpha x + 2020 = 0$ are all integers?

**Solution**

The roots will be factors of 2020.

$2020 = 2^2 \times 5 \times 101$

The +ve integer factors of 2020 in pairs are:

(1, 2020) (2, 1010) (4, 505) (5, 404) (10, 202), (20, 101)

But negative values of these will also be factors.

So, there are 12 values of $\alpha$ for which the roots are integers.

**Question 4.**

Five perfect squares have a mean of 59, a median of 4, and a mode of 1. The second largest of the numbers is a two-digit number 'ab'. What is a – b?

**Solution**

4 is the median and therefore the third of the numbers in ascending order. 1 occurs at least twice so it is both the first and second number.

If $x^2$ and $y^2$ are the remaining numbers, then $(6 + x^2 + y^2) = 5 \times 59 = 295$.

So $x^2 + y^2 = 289 = 17^2$.

x, y and 17 therefore form a Pythagorean Triple, the integer sides of a right-angled triangle.

$289 = 64 + 225 = 8^2 + 15^2$

So, ab = 64 and a – b = 2.