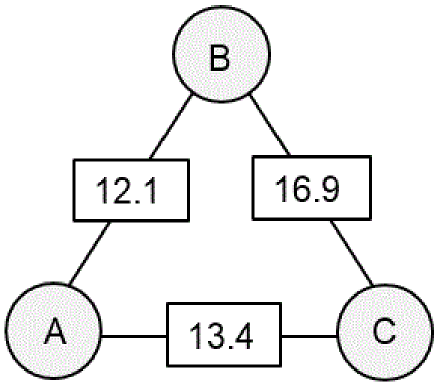
Week Thirty-five problems and solutions

Question 1.

In the below figure, called an Arithmagon, the numbers that appear in the boxes are the sum of the numbers in the circles on each side of them. Find the numbers that belong in each circle?



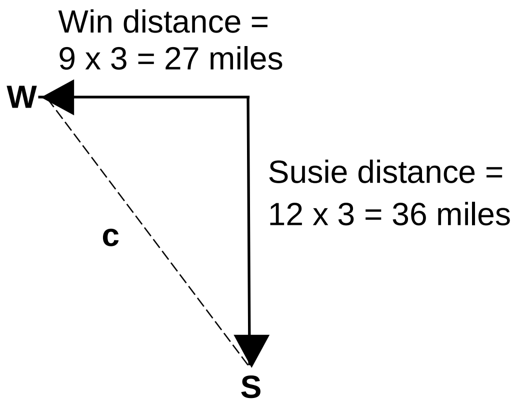
Solution

A + B = 12.1
A + C = 13.4
B + C = 16.9
Adding these we get 2(A + B + C) = 12.1 + 13.4 + 16.9 = 42.4 i.e.
A + B + C = 21.2
A = (A + B + C) – (B + C) = 21.2 – 16.9 = 4.3
B = (A + B + C) – (A + C) = 21.2 – 13.4 = 7.8
C = (A + B + C) – (A + B) = 21.2 – 12.1 = 9.1

Question 2.

Susie and Win both enjoy cycling and agreed to meet up for lunch in a café. After lunch they left at the same time, to cycle in different directions and speeds. Susie headed South at 12 m.p.h. while Win headed West at 9 m.p.h. They are both radio amateurs and arranged to make radio contact after 3 hours. If they are no more than 50 miles apart by then this may be possible. Will they be able to make radio contact?

Solution



Using Pythagorean theorem, $a^2 + b^2 = c^2$, or $(27)^2 + (36)^2 = c^2$

So, $c^2 = 729 + 1296 = 2025$

$c = \sqrt{(2025)} = 45$ miles

This is less than 50 miles, therefore, radio contact may be possible.

Question 3.

The sum of four different prime numbers is prime. The sum of one pair is prime; and the sum of one triple is also prime. What is the smallest possible sum of four prime numbers with these properties?

Solution

One of the four prime numbers must be 2, as otherwise the sum of the prime numbers would be even and hence not a prime.

The triple that sums to a prime must consist of three odd primes and will sum to at least 3 + 5 + 7 = 15, or 17 with the fourth prime 2.

However, the pair (15, 17) are not both primes.

The lowest pairs above this that are both primes are (17, 19) and (29, 31)

No solution exists for (17,19)

A solution can be found for (29,31) with 2 + 5 + 7 + 17 = 31.

Question 4.

$0 < b < a$ and $a^2 + b^2 = 6ab$. What is the value of $(a + b)/(a – b)$?

Solution

$(a + b)^2 / (a – b)^2$

$= (a^2 + 2ab + b^2) / (a^2 – 2ab + b^2)$

$= (6ab + 2ab) / (6ab – 2ab)$

$= 8ab/4ab$

$= 2$

Hence $(a + b) / (a - b) = \pm \sqrt{2}$

But $a - b > 0$ and both a and b are +ve

Therefore $(a + b) / (a - b) = + \sqrt{2}$