Week Seventeen Solutions

Question 1.

The toll rates for crossing a bridge are £6 for a car and £10 for a lorry. During a two-hour period, a total of 187 cars and lorries crossed the bridge, and the total collected in tolls was £1,262. How many cars and how many lorries crossed the bridge during the two hours?

Solution

Suppose that the number of cars crossing the bridge was C and the number of lorries was L

Then

C + L = 187

6C +10L = 1262

Multiplying the first equation through by 10 we get

10C + 10L = 1870

Subtracting the second equation from this third equation gives

4C = 608 so C = 152

So, given C = 152, the first equation implies that L = 187 – 152 = 35

The number of cars crossing the bridge was 152 and the number of lorries 35.

Question 2.

A typical image taken of the surface of Mars by a camera is 11.2 gigabits in size. (1 gigabit equals 1024 megabits). A tracking station on Earth can receive data from the spacecraft at a data rate of 3 megabits per second for a maximum of 11 hours each day. What is the maximum number of complete images that the tracking station could receive from the camera each day?

Solution

Each image size = 11.2 Gb x 1024 Mb/Gb = 11468.8 Mb

Transfer time/image = image size / data rate = 11468.8 / 3 = 3822.9 sec.

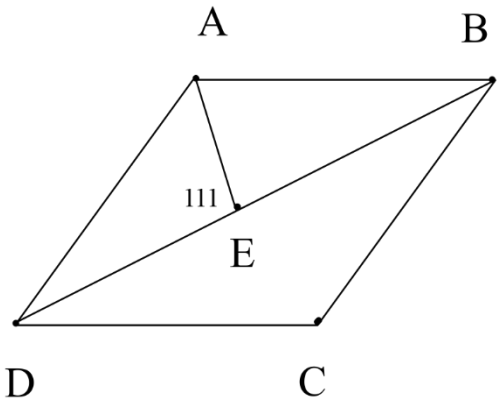
Time in a 11 hour day = 11 x 60 x 60 = 39600 seconds

Maximum number of images received /day = 39600 / 3822.9 = 10.36 to 2 decimal places

So maximum number of complete images per day is 10

Question 3.

The diagram below shows a rhombus ABCD and within it an isosceles triangle ABE with BA = BE. If < AED = 111°, what is < DAE?



Solution

Since Δ ABE is isosceles and < BEA = 69°, < ABE = 180 – 2 x 69 = 42°

ABCD is a rhombus so Δ ADB is isosceles and hence < ADB = < ABD = 42°

In Δ ADE, < DAE = 180 – 111 – 42 = 27°

Question 4.

(From 6th century China).

A cock is worth 5 coins; a hen worth 3 coins; and three chickens are worth one coin. If 100 animals are sold for 100 coins what are their composition between cocks, hens, and chickens?

Solution

Let A be the number of cocks, B the number of hens, C the number of chickens.

Then from the numbers $A + B + C = 100$ (a)

and from the costs $5A + 3B + C/3 = 100$ (b)

$15A + 9B + C = 300$ (3 x b)

$$14A + 8B = 200 \quad (3 \times b - a)$$

$$7A + 4B = 100$$

Now A and B are whole numbers, so 100 - 4B is divisible by 4 and 7.

Therefore, 7A must be 84 or 56 or 28; with 4B correspondingly 16 or 44 or 72.

So, using the fact that A + B + C = 100, the possibilities for (A, B. C) are
(12, 4, 84); (8, 11, 81); (4, 18, 78)